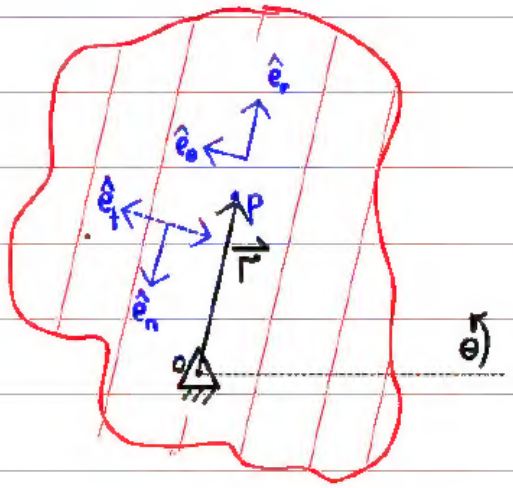
Meriam 5/2: Fixed-Axis Rotation (contd.)

$$\vec{v}_P = \vec{v}_O + \vec{v}_{P/O}$$

$$\boxed{\vec{v}_P = 0 + |\omega\, r|\hat{e}_t} \leftarrow$$

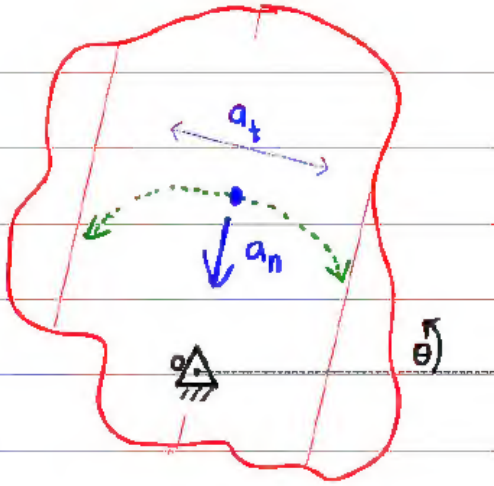
(or $\vec{v}_P = \vec{\omega} \times \vec{r}$)



$$\vec{a}_P = \vec{a}_O + \vec{a}_{P/O}$$
$$= 0 + (a_{P/O})_n\, \hat{e}_n \;+/-\; (a_{P/O})_t \hat{e}_t$$

$$\boxed{\vec{a}_P = 0 + \omega^2 r\, \hat{e}_n \;+/-\; \alpha r \hat{e}_t}$$

(or $\vec{a}_P = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$)



★ We recommend that you use the formulas contained in the blue boxes ☐ above for this 2D course to get the magnitude of a_n, a_t ★

Then get the direction from the diagram.

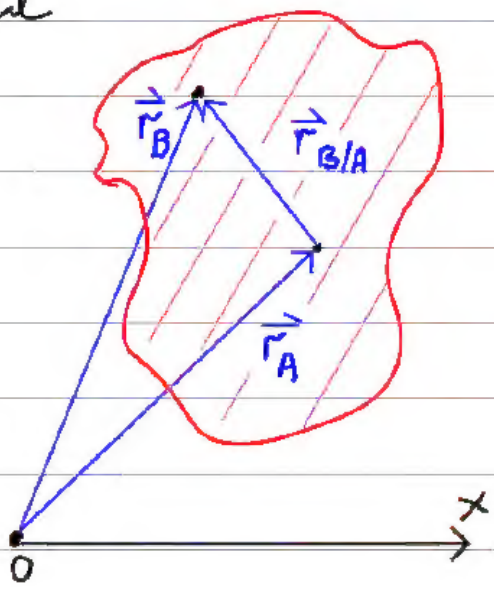
General Plane Motion

Consider the acceleration of point P when there is no fixed axis O.

e.g. the kinematics of a car wheel

For any 2 points A & B,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$



$\frac{d}{dt}$ →

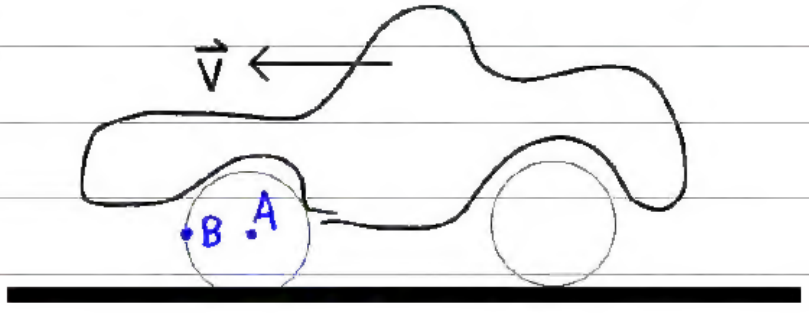
$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} \quad \bigstar$$

(Up to now, we have only considered the relatively simple case of A = a fixed point O, yielding

$$\vec{V}_B = \cancel{\vec{V}_O}^{\,0} + \vec{V}_{B/O} = |\omega|\,|\vec{r}|_B\,\hat{e}_t$$

How can we use eqn ★ to solve for $\vec{V}_B$ for the general case when there is no fixed axis?

ie, $\vec{V}_A \neq 0$?



Answer

1. Pick a "convenient" reference point "A".

Note: B cannot get any closer or further from A.

→ To an observer standing at A, point B appears to be going in a circle around A.

2/ We now attempt to draw the vector triangle described by

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} \quad \bigstar$$

Use the following key facts to help:

(i) If you choose point A wisely, then maybe you know $\vec{V}_A$

(ii) $\vec{V}_{B/A} \perp \vec{r}_{B/A}$

(iii) $|\vec{V}_{B/A}| = |\omega r_{AB}|$

3/ Complete the vector triangle to find $\vec{V}_B$.

Note: If the object is not rotating

→ $\omega = 0$

→ $|\vec{V}_{B/A}| = |\omega^{\circ} r_{AB}| = 0$

→ $\vec{V}_B = \vec{V}_A + \vec{V}^{\circ}_{B/A} = \vec{V}_A$

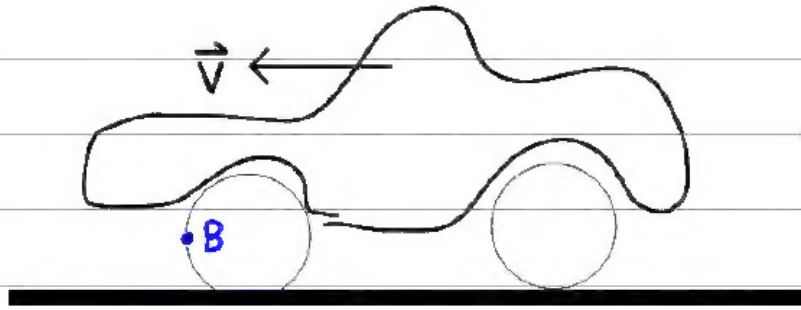
→ All points have the same velocity → pure translation

Eg.

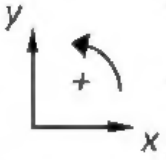
car velocity $= -20\ m/s\ \hat{i}$

wheel radius $= 35\ cm$

wheel $\omega = +57.14\ s^{-1}$



Find $\vec{V}_B$

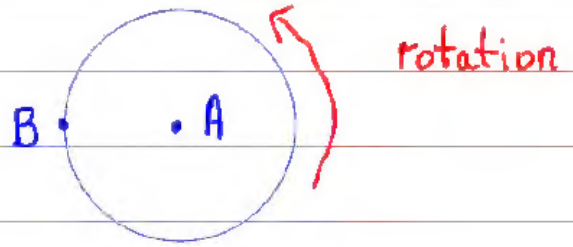


Answer

Choose 'A' as a reference point

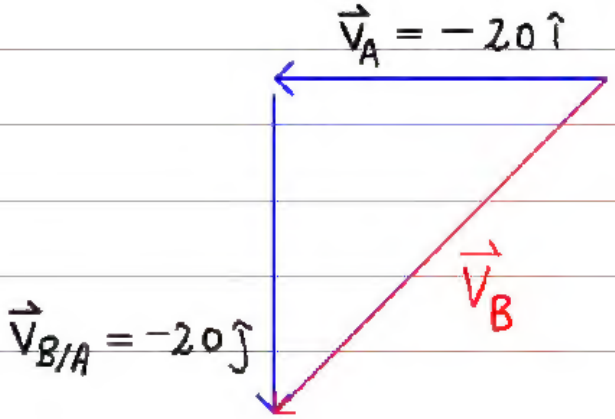
$\vec{V}_A = -20\ m/s\ \hat{i}$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$



$\vec{V}_{B/A} \perp \vec{r}_{B/A}$

$$|\vec{V}_{B/A}| = |\omega r_{AB}|$$
$$= (57.14)(0.35)$$
$$= 20\ m/s \downarrow$$



$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = -20\hat{i} - 20\hat{j}\ m/s$$

Get vector directions from the diagram

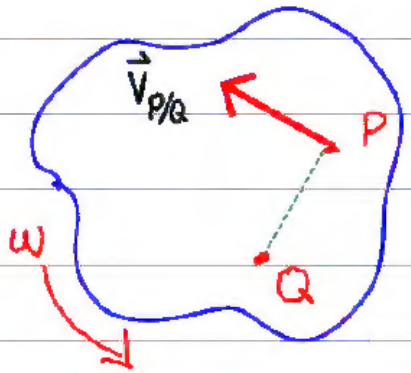
On a solid body, we can get the velocity of any point B if we know:

(i) geometry of the solid body
(ii) velocity of any point A : $\vec{V}_A$
(iii) ω

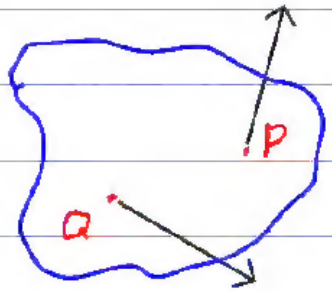
But ω may be unknown. How can we determine ω?
Suppose we know the velocities of any 2 points, P & Q :

$$\rightarrow \quad |\vec{V}_{P/Q}| = |\vec{V}_P - \vec{V}_Q| = |\omega \vec{r}_{P/Q}|$$

$$|\omega| = \frac{|\vec{V}_P - \vec{V}_Q|}{r_{PQ}} \quad \star$$



- magnitude of ω
- get direction of ω from the diagram.



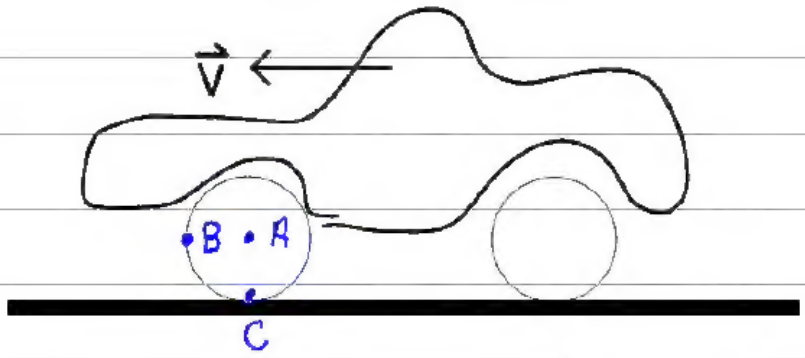
Remember: The value of ω that you get from eqn $\star$ applies to the entire body, and not just to points P & Q

Eg.

car velocity = -20 m/s $\hat{i}$

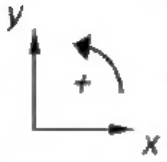
wheel radius = 35 cm

~~wheel $\omega = +57.14\ s^{-1}$~~



Find $\vec{V}_B$

Answer:

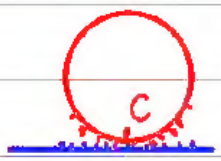


We know $\vec{V}_A = -20$ m/s $\hat{i}$

We need the velocity of a second point to get ω.

If the wheel is not skidding → $\vec{V}_C = \vec{V}_{road} = 0$

→ $|\omega| = \dfrac{|\vec{V}_A - \vec{V}_C|}{r_{AC}} = \dfrac{|-20\hat{i} - 0|\ m/s}{0.35\ meters} = 57.14\ s^{-1}$



Get the direction of ω from the diagram

→ $\omega = +57.14\ s^{-1}$ or $57.14\ s^{-1}\ \curvearrowright$

Then proceed as on page 20(d) to get

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = -20\hat{i} - 20\hat{j}\ m/s$$

magnitude $|\omega r_{AB}|$ direction $\perp r_{AB}$

Summary

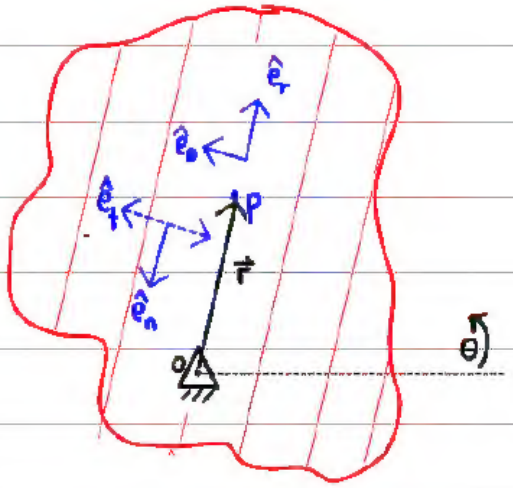
Fixed-Axis Rotation:

$$\vec{v}_P = 0 + |\omega r|\hat{e}_t$$

(or $\vec{v}_P = \vec{\omega} \times \vec{r}$)

$$\vec{a}_P = 0 + \omega^2 r \hat{e}_n \quad +/- \alpha r \hat{e}_t$$

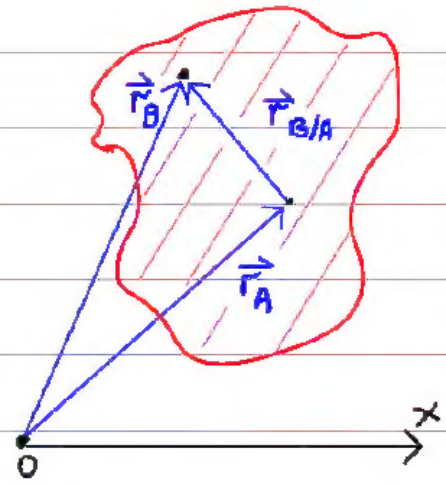
(or $\vec{a}_P = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$)



General Plane Motion:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

magnitude ωr_{AB} direction $\perp r_{AB}$



where $$|\omega| = \frac{|\vec{v}_P - \vec{v}_Q|}{r_{PQ}}$$

(or $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$)

Next Lecture

- Read Meriam 5/3 – 5/5
- Solved Problems & Engineering Applications